IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
PP 45-48

Gracefulness Of Contract Supersubdivision Of Some Graphs

[1]A. Anand Ephremnath, [2] S. Nadhiya, [3] G. Vijayalakshmi, [4] Hema

[1]*Department of Mathematics, Surya Group of Institutions School of Engineering and Technology, Vikiravandi, Villupuram – 605652,*
[2]*Department of Electronics and Communication Engineering, Surya Group of Institutions School of Engineering and Technology, Vikiravandi, Villupuram – 605652, India*
[3]*Department of Electronics and Communication Engineering, Surya Group of Institutions School of Engineering and Technology, Vikiravandi, Villupuram – 605652, India*
[4]*Department of Computer Science Engineering, Surya Group of Institutions School of Engineering and Technology, Vikiravandi, Villupuram – 605652, India*

Abstract: *We investigate some new results for graceful labeling of graph. In this paper we define an arbitrary contract supersubdivision of k-block paths in the increasing length and furthermore we prove that such graph is graceful.*
Keywords: *Graceful labeling; Arbitrary Supersubdivision; Contract Supersubdivision.*

I. Introduction

We begin with simple, finite, undirected and connected graph $G = (p, q)$. A *graceful labeling* of G is an injection from the set of its p vertices to the set $\{0, 1, 2, \ldots, q\}$ such that the values of the edges are all integers from 1 to q, the value of an edge being the absolute value of the difference between the integers attributed to its end vertices.

Sethuraman and Selvaraju [7] have introduced a new method of construction called *supersubdivision* of a graph and they proved that every supersubdivision of a path is graceful and every cycle has some supersubdivision that is graceful. After that Sekar and Ramachandran [6] proved that arbitrary supersubdivision of disconnected graph is graceful. Ambili and Singh [1] defined an *arbitrary strong supersubdivision* of a graph and they proved that arbitrary strong supersubdivision of paths, cycles and stars are graceful.

Pemmaraju and Skiena [5] used the concept of contracting the vertices and they are limited to studying time complexity of the isomorphism problem. Ivanco and Semanicova [3] proved that every 3-regular triangle-free supermagic graph has an edge such that the graph obtained by contracting that edge is also supermagic and the graph obtained by contracting one of the edges joining the two n-cycles of $C_n \times K_2$ ($n \geq 3$) is supermagic. For detail survey on graceful labelling one can refer to Gallian [4].

Elumalai and Anand Ephremnath [2] have introduced a new method of construction called an arbitrary contract supersubdivision and furthermore they prove that an arbitrary contract supersubdivision of disjoint union of paths and cycles are graceful.

In this paper we define an arbitrary contract supersubdivision of k-block paths in the increasing length and furthermore we prove that such graph is graceful.

Definition 1.1 : Let G be a graph with q edges. A graph H is called a supersubdivision of G if H is obtained from G by replacing every edge ei of G by a complete bipartite graph $K_{2,\,mi}$ for some mi , $1 \leq i \leq q$ in such a way that the end vertices of each ei are merged with the two vertices of 2-vertices part of $K_{2,\,mi}$ after removing the edge ei from graph G. A supersubdivision H of G is said to be an arbitrary supersubdivision of G if every edge of G is replaced by an arbitrary $K2,m$ (m may vary for each edge arbitrarily).

Definition 1.2 : The contraction of a pair of vertices vi and vj of a graph produces a graph in which the two nodes vi and vj are replaced with a single node v such that v is adjacent to the union of the nodes to which vi and vj were originally adjacent. In vertex contraction, it doesn't matter if vi and vj are connected by an edge; if they are, then the edge disappear when vi and vj are contracted. Also vi and vj are from two components of a disconnected graph.

Definition 1.3 :

Let G be the disconnected graph with path components $P_{j+1} = v_1^j, v_2^j, \ldots, v_{j+1}^j$, $1 \leq j \leq k$ .A graph H is called an arbitrary contract super subdivision of k-block paths in the increasing length with of G if H is obtained from G by replacing every edge $e^j = v^j v^j$,$1 \leq j \leq k$, $1 \leq i \leq j$ of G by a complete bipartite graph $K\,2\,,\,m\,j$ for some m^j *I i i* +1 *I* ($m^j \geq 2$ *if* $j = k$, $1 \leq i \leq j - 1$ and $1 \leq i, j \leq k - 1$ and also $m^j \geq 3$ *if* $2 \leq j \leq k - 1$, $1 \leq i \leq j - 1$) *I I* in

such a way that the end vertices of each e_ij are merged with the two vertices of 2-vertices part of K_j after removing the edge e_i^j from graph G, then labeling the m_i^j vertices part of $K\,2\,,\ m_i\,2\,,\ m_i\,2\,,\ m_i\,1 \le j \le k\,,\ 1 \le i \le j-1$ by N^j, N^j, N^j, N^j and contracting the vertices N^j and $i,i,\,2\,i,\,3\ i,\ m_i\,j\ i,\ m_i\ _N\,j{+}1$ (*say* W^j) , $1 \le j \le k-1,\ 1 \le i \le j$, so that H is a connected graph. [For example refer $i,1\ i$

Figure-1

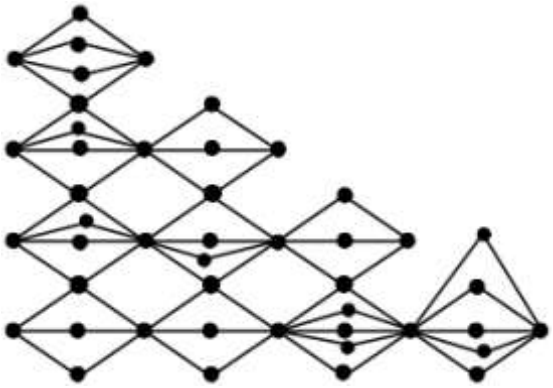

fig 1 : arbitrary contract supersubdivision of 4-block paths in the increasing length

II. Main Results

In this section we discuss about the gracefulness of the graph obtained by arbitrary contract supersubdivision of k-block paths in the increasing length.

Theorem 2.1 : The graph obtained by arbitrary contract supersubdivision of k-block paths in the increasing length is graceful.

Proof : Let G bethe disconnected graph with $k$path components $P_{j+1} = v^j_1, v^j_2, \ldots, v^j_{j+1}$, $1 \le j \le k$, let H be an arbitrary contract supersubdivision of G as described in definition 1.3 and let H contains M edges and N vertices.

Note that the two vertices of the 2-vertices part of K_j , $1 \le i \le j$ get the labels v^j_i and v^j_{i+1} .

Now welabel the non-contracting vertices of K_j by $N_i^j, N_i^j - 2, N_i^j - 4, \ldots$ and the contraction vertices have the labels W^j_i , $1 \le j \le k-1,\ 1 \le i \le j$ as shown Figure-2.

Let

$$M_j = \sum_{i=1}^{j} m_i^j\ ,\ 1 \le j \le k$$

$$E_1 = M = 2\left\lceil \frac{M_l}{\sum_{l=1}^{k} l} \right\rceil$$

$$E_j = M - 2\sum[M_l - l] - 2[j-1],\ 2 \le j \le k$$

l=1

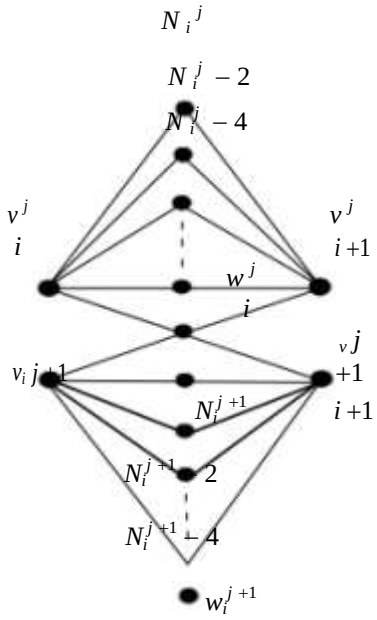

Fig 2

Define

$$v_i^j = (i-1) + j\,(j-1),\ 1 \le j \le k,\ 1 \le i \le j+1$$

$$w_i^j = (1-i) + M - 2\sum_{l=1}^{j}[M_l - l],\ 1 \le j \le k-1,\ 1 \le i \le j$$

$$N_i^j = \begin{cases} E_j - 2\sum_{l=1}^{i-1} l + j + 3(i-1), & if \quad j = 1\ \&\ k,\ \ 1 \le i \le j \\ E_{j-2} + 2\sum_{l=1}^{i-1} l + 5(i-1), & if \quad 2 \le j \le k-1,\ 1 \le i \le j \end{cases}$$

It is clear from the above labeling that all the vertices of *H* have distinct labels, that is *H* has an injection from the set of its *N* vertices to the set {0, 1, 2, . . . , *M* } and the values of the edges of *H* are all integers from 1 to *M*, so *H* is graceful.

III. Conclusion

In this paper we discussed about the gracefulness of the graph obtained by arbitrary contract supersubdivision and we proved that the graph obtained by arbitrary contract supersubdivision of *k*-block paths in the increasing length is graceful.

Example : Graceful labeling of the graph which is in Fig-1

76

74

0 72 1

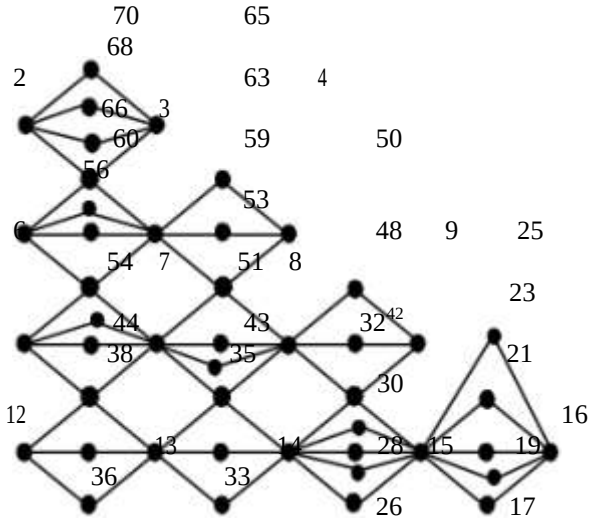

Fig 3 : Some arbitrary contract supersubdivision of 4-block paths in the increasing length

References

[1] Ambili B. and Singh G. S., Graceful labeling of arbitrary strong supersubdivisions of graphs, *Graph Theory Notes N. Y., 61* (2011) 31-36. http://gtn.kazlow.info/GTN61.pdf

[2] Elumalai. A and Anand Ephremnath. A, Arbitrary contract supersubdivision of paths and cycles are graceful, Proceedings of International Conference on Mathematics and its Application (ICMAA-2014), Anna University, Villupuram, Vol 2, pp. 801-808.

[3] Ivanco J. and Semanicova A., Some constructions of supermagic non-regular graphs, *Australas. J. Combin.*, 38 (2007) 127-139.

[4] Joseph A. Gallian,, Dynamic Survey of Graph Labeling, *The Electronic Journal of Combinatorics*, *16*, (2013).

[5] Pemmaraju S. and Skiena S. "Contracting Vertices." §6.1.1 in *Computational Discrete Mathematics: Combinatorics and Graph Theory with Mathematica.* Cambridge, *England: Cambridge University Press*, pp. 231-234, 2003.

[6] Sekar C. and Ramachandran V., Graceful labeling of arbitrary supersubdivision of disconnected graph, *Ultra Scientist Vol. 25(2)A*, 315-318 (2013).

[7] Sethuraman G. and Selvaraju P., Gracefulness of arbitrary supersubdivision of graphs, *Indian J. Pure Appl. Math.*, 1059-1064 (2001).